Department of Pesticide Regulation

M E M O R A N D U M

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SUBJECT: WHY WORKER HEALTH AND SAFETY BRANCH USES ARITHMETIC MEANS IN EXPOSURE ASSESSMENT

The Worker Health and Safety (WHS) Branch generally uses the arithmetic mean in exposure assessments whenever a measure of central tendency is needed. This memorandum summarizes the reasons for this practice.

While extremely high exposures are low-probability events, they do occur, and the arithmetic mean appropriately gives them weight in proportion to their probability. In contrast, the geometric mean gives decreasing weight as the value of the exposure increases, and the median gives no weight to extreme exposures.

It is sometimes argued that extremely high values represent measurement errors. Parkhurst (1998) points out that while measured values are subject to sampling and analytical error, there is no reason to suppose that the errors are biased high, and therefore no reason to downplay large values. Rather than use a statistic that discounts extreme values, WHS discards a measured value of any magnitude if there is strong reason to suspect its validity.

Both human exposures and environmental concentrations are in most cases either known or assumed to be lognormally distributed. Regardless of the shape of the underlying distribution, WHS uses the arithmetic mean, rather than the geometric mean or the median. Although it can be argued that the latter statistics better indicate the location of the center of a skewed distribution, it is not the location that is of interest in exposure assessment, but the expected magnitude of exposure.

Environmental residues

Human exposure to a contaminant in an environmental medium can be estimated from the concentration of contaminant in the medium and the rate of transfer of contaminant from the medium to people. The spatial and temporal extent of medium to which people are or might be exposed must be defined so that the appropriate environmental concentration can be measured. For example, to use dislodgeable foliar residue (DFR) measurements to estimate one-day dermal exposure of fruit thinners, the DFR measure should represent the foliage in the area that a worker could thin during a defined time period (e.g., the 8-hr period starting 72 hours after application). That is, the DFR measure should represent both the appropriate spatial area and the appropriate time interval. For estimating inhalation exposure, the target spatial area is the region a person could be expected to be within during the time period of interest. The air concentration measure should represent the air in that region during that time interval.

The best estimate of the average mass of residue per unit of environmental medium is the arithmetic mean concentration (Parkhurst, 1998). Conceptually, a perfect (apart from analytical error) DFR measurement representing the foliage a worker could contact during some time interval would be obtained by collecting *all* of that foliage and submitting it to DFR analysis as one huge sample. The total mass of residue dislodged divided by the total foliar surface area would be, in a sense, the "true" DFR value. If, instead of analyzing all the foliage as one sample, it were divided into portions that were analyzed separately, the arithmetic mean of the portion DFRs would be identical, apart from analytical error, to the value obtained from the one huge sample. Further, if instead of analyzing all the portions, a number of portions were randomly selected and analyzed, their arithmetic mean would have the same expected value. Taking the arithmetic mean of *n* sample measurements is mathematically equivalent to compositing all *n* samples and measuring the concentration of the mixture.

The same logic applies to estimating concentration in any environmental medium. For air concentrations, the arithmetic mean of time-weighted samples from spatially randomized samplers has the expected value of the concentration in all of the air in the region over the time interval.

Short-term exposures

Short-term dermal exposure is measured with patches or whole-body dosimeters worn during a short exposure period, the most typical for worker exposure being one work day. The objective is to estimate the total mass (μg) of active ingredient deposited on a person's skin or clothing. When patches are used, total exposure is calculated by multiplying the average $\mu g/cm^2$ of the patches by the total surface area of the body or body part. The arithmetic mean is the appropriate average to use, as the following explains. A whole-body dosimeter comes as close as we are able to capturing the total potential dermal dose. If the whole-body dosimeter were cut into small pieces and each piece analyzed, the sum of the masses on the individual pieces would equal the total mass on the whole dosimeter (apart from measurement error). The arithmetic mean $\mu g/cm^2$ of the pieces, multiplied by the total surface area of the pieces, would also equal the mass on the whole dosimeter. The geometric mean $\mu g/cm^2$ of the pieces, multiplied by the total surface area of the pieces, is less than the total mass, except in the case that every piece has the same value. In general, the more variable the pieces, the more the geometric mean underestimates total mass. A simple numeric illustration is attached (Attachment 1). The same reasoning applies when patches are used. In that case, the arithmetic mean $\mu g/cm^2$ of patches multiplied by total body surface area will differ from the true total mass due to sampling error, but its expected value is the same.

The Pesticide Handlers Exposure Database (PHED) calculates average daily exposure over persons by calculating average exposure per body part, then summing the body-part averages to get average total exposure. Mathematically, the sum of arithmetic mean body-part exposures is equivalent to arithmetic mean total-body exposure. This is not true, however, for the geometric mean. A numeric illustration is attached (Attachment 2). In fact, it is not possible to construct the geometric mean total-body exposure from the geometric mean body-part exposures.

Long-term exposures

It is not always clear what characteristic of seasonal or longer exposure is important. It is probably not the same characteristic for every pesticide. In one case, only the highest exposure during a period might matter, in another, the number of times a given exposure level is exceeded, or the number of consecutive days of exposure. As a general approach, however, it is intuitively compelling that the total mass exposure received during the interval would be important. If daily exposure in μg/day were measured every work day of a work season, the sum of a person's daily measurements would be his total measured seasonal exposure. The same value would be obtained by multiplying the person's arithmetic mean daily exposure by the number of days worked. The geometric mean daily exposure does not give back the total exposure. The geometric mean is reflective of an exposure near the median; as such, it approximately represents the exposure day that is higher than half the days and lower than half the days. If the total mass exposure is important, however, the *magnitudes* of all the daily exposures must be represented, as they are in the arithmetic mean. When a sample of exposure days is monitored, rather than the whole season, the arithmetic mean μg/day multiplied by the total number of work days in the interval will differ from the true total mass due to sampling error, but its expected value is the same.

Although exposure assessors do not ordinarily have repeated measurements on individuals and must rely on a sample of measurements from different individuals, the same argument for using the arithmetic mean applies.

Computational use of the geometric mean

The above arguments notwithstanding, the geometric mean can be used as a mathematical convenience in calculating percentiles. The p^{th} percentile of a lognormal distribution can be estimated by either of two formulas that give identical results:

(i) (Geometric Mean) * (Geometric Standard Deviation)^$t_{(p,n-1)}$.

(ii) Antilog{(Arithmetic Mean of logs) + (Arithmetic Standard Deviation of logs)*t(p,n-1)}.

The formula using the geometric statistics is computationally simpler and for that reason may be preferred.

Reference

Parkhurst, D.F. 1998. Arithmetic versus geometric means for environmental concentration data. *Environmental Science and Technology News.* Feb. 1.

Attachments

Illustration of calculating total deposition to a body part (of one person) using the average of patches on the body part

Imagine a hypothetical body party 100 cm^2 in area and completely covered by 10 equal-sized patch dosimeters. The "true" total mass of residue on the body part is the total of the masses on the 10 patches. The table below represents the data for one person, whose actual total deposition is 86 μg . When this total is estimated by multiplying the average $\mu g/cm^2$ of the 10 patches times the known total area (100 cm^2), using the arithmetic mean $\mu g/cm^2$ gives the correct total, while using the geometric mean $\mu g/cm^2$ gives a value that is too low. The percentage by which the GM-based estimate is too low increases as the between-patches coefficient of variation increases.

Patch #	ug	cm2	ug/cm2
1	3	10	0.30
2	3	10	0.30
3	3	10	0.30
4	4	10	0.40
5	4	10	0.40
6	5	10	0.50
7	8	10	0.80
8	12	10	1.20
9	14	10	1.40
10	30	10	3.00
Total	86	100	0.860

Arithmetic mean μg /cm2	0.860
Geometric mean μg /cm2	0.622

Total ug estimated from average of patches:

AM* total area=	86	ug
GM* total area=	62	ug
Estimate using GM =	72	% of actual total

When a sample of patches from the body part is used, rather than covering it completely, the arithmetic-mean based estimate will not be identical to the true total. However, its expected value is the true total value, while the expected value of the geometric-mean based estimate is less than the true total.

Illustration of calculating average total deposition to the body as the sum of body-part averages (as done by the PHED)

The PHED calculates mean exposure per body part, then sums body-part means to get mean total-body exposure. The table below has hypothetical data on five body parts for 10 individuals. The cell entries are the total measured (or estimated) μg on the total surface area of each body part for each person. The last two columns give the geometric and arithmetic means of the 10 measurements for the body part. The last row of the table gives actual total-body μg for each person; the last two entries of the last row are the means of the row entries, i.e., they are the true means of total-body exposure calculated from 10 peoples' total-body measurements.

	Total μg on body part										Body-Part Means	
Person	1	2	3	4	5	6	7	8	9	10	Geometric	Arithmetic
Chest	20	52	54	56	58	60	80	90	100	150	64.2	72.0
Back	20	10	10	30	30	3	22	45	45	3	15.3	21.8
Upper arm	30	30	24	30	10	30	32	50	36	80	31.3	35.2
Forearm	30	50	30	50	20	30	87	50	60	70	43.6	47.7
Hand	90	150	80	80	70	70	400	180	400	200	137.6	172.0
Total body	190	292	198	246	188	193	621	415	641	503	309.8	348.7

Sum of body-part geometric means = 292.0
Sum of body-part arithmetic means = 348.7

PHED summary output only gives means per body part and mean total-body exposure calculated as the sum of the body-part means. The body-part arithmetic means sum to the actual total-body arithmetic mean, but the body-part geometric means do not sum to the total-body geometric mean. It can be shown by simulation that the sum of body-part geometric means consistently underestimates true total-body geometric mean.